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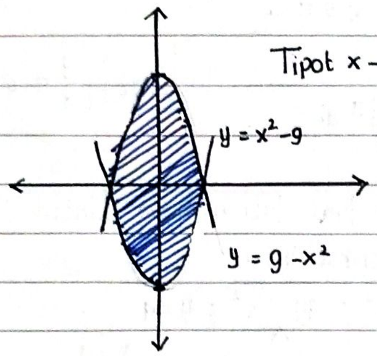
KPB (B)

8/11/2023

12. Hitung luas bidang datar yang dibatasi oleh: $y = 9 - x^2$ dan $y = x^2 - 9$

▶ Grafik $y = 9 - x^2$
merupakan kurva parabola yang menghadap ke bawah dengan puncak $(0,9)$.

▶ Grafik $y = x^2 - 9$
merupakan kurva parabola yang menghadap ke atas dengan puncak $(0,-9)$



Tipot $x \longrightarrow y = 0$

$x^2 = 9$

$x = \pm 3$

Untuk menentukan luasan seluruh daerah yang diarsir warna biru, ambil daerah kuadran I saja. Karena simetris, maka kalikan 4 luas pada kuadran I, akan diperoleh luas total daerah warna biru.

$$L_I = \int_0^3 \int_0^{9-x^2} dy\, dx$$

$$= \int_0^3 y \Big]_0^{9-x^2} dx$$

$$= \int_0^3 (9 - x^2)\, dx$$

$$= 9x - \frac{1}{3}x^3 \Big]_0^3$$

$$= \left[9(3) - \frac{1}{3}(3)^3\right] - \left[9(0) - \frac{1}{3}(0)^3\right]$$

$$= \left(27 - \frac{27}{3}\right) - 0$$

$$= 27 - 9$$

$$= 18$$

$$L_{total} = 4 \times L_1$$

$$= 4 \times 18$$

$$= 72 \text{ satuan luas}$$

Jadi, luasan bidang datar tersebut adalah 72 satuan luas.

21). Hitung Volume ellipsoida $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

▶ Di bidang XYO

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \longrightarrow$ elips dengan pusat (0,0) dgn panjang sumbu di sumbu x adalah a dan panjang sumbu di sumbu y adalah b.

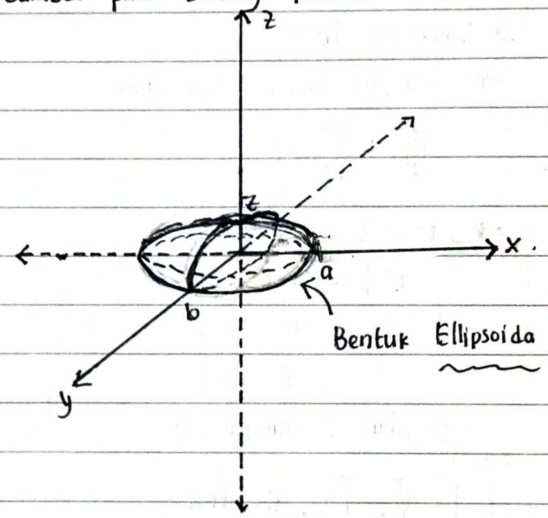
▶ Di bidang XOZ

$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1 \longrightarrow$ Elips dengan pusat (0,0) dgn panjang sumbu di sumbu x adalah a dan panjang sumbu di z adalah c

▶ Di bidang OYZ

$\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \longrightarrow$ Elips dengan pusat (0,0) dgn panjang sumbu di y adalah b dan panjang sumbu di z adalah c.

※ Gambar pada bidang koordinat XYZ



Untuk menentukan Volume total, hitung saja Volume pada sumbu z positif lalu kalikan dua untuk memperoleh Volume total.

Sumbu $z \rightarrow$ BA : $\frac{z^2}{c^2} = 1 - \left[\frac{x^2}{a^2} + \frac{y^2}{b^2}\right]$

$$\frac{z}{c} = \sqrt{1 - \left[\frac{x^2}{a^2} + \frac{y^2}{b^2}\right]}$$

$$z = c\sqrt{1 - \left[\frac{x^2}{a^2} + \frac{y^2}{b^2}\right]}$$

BB = 0

$$\iint_A \int_0^{c\sqrt{1-\left[\frac{x^2}{a^2}+\frac{y^2}{b^2}\right]}} dz\, dy\, dx$$

$$\iint_A c\sqrt{1-\left[\frac{x^2}{a^2}+\frac{y^2}{b^2}\right]}\, dy\, dx$$

Misalkan : $\frac{x}{a} = r\cos\theta \qquad \frac{x^2}{a^2} = r^2\cos^2\theta$

$\frac{y}{b} = r\sin\theta \qquad \frac{y^2}{b^2} = r^2\sin^2\theta$

※ Jacobian

$$J = \frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix}$$

$$= \begin{vmatrix} a\cos\theta & -ar\sin\theta \\ b\sin\theta & br\cos\theta \end{vmatrix}$$

$$= abr\cos^2\theta - \left[-abr\sin^2\theta\right]$$

$$= abr\left[\cos^2\theta + \sin^2\theta\right]$$

$$= abr$$

※ Penentuan batas.

⇒ Daerah dibatasi oleh garis :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

⇒ $r^2\cos^2\theta + r^2\sin^2\theta = 1$

$r^2\left[\cos^2\theta + \sin^2\theta\right] = 1$

$r^2 = 1$

$r = 1$

θ dari 0 sampai 2π

$$\int_0^{2\pi}\int_0^1 c\sqrt{1-r^2}\,.\, abr\, dr\, d\theta$$

$$= abc \int_0^{2\pi}\int_0^1 r\sqrt{1-r^2}\, dr\, d\theta$$

misal : $k = 1-r^2$ $\qquad$ BA : $1-1 = 0$

$dk = -2r\,dr$

$\frac{-dk}{2} = r\,dr$ $\qquad$ BB : $1-0 = 1$

$$= abc \int_0^{2\pi}\int_1^0 \sqrt{k}\,.\, -\frac{dk}{2}\, d\theta$$

$$= -\frac{abc}{2}\int_0^{2\pi} \frac{2}{3}k\sqrt{k}\Big]_1^0 d\theta$$

$$= \frac{-abc}{2}\cdot\frac{2}{3}\int_0^{2\pi}(-1)\, d\theta$$

$$= \frac{abc}{3}\cdot\theta\Big]_0^{2\pi}$$

$$= \frac{abc}{3}\cdot 2\pi$$

$$= \frac{2\pi abc}{3}$$

$V_{tot} = 2 \times V_1$

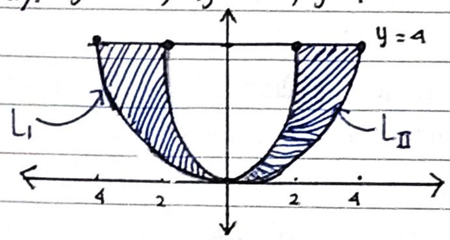
$= 2 \times \frac{2\pi abc}{3}$

$= \frac{4\pi}{3}abc$

Jawab = $\frac{4}{3}\pi abc$ satuan volume

22). Hitung luas daerah yang dibatasi oleh kurva-kurva berikut ini :

a). $y = x^2$; $4y = x^2$; $y = 4$



▶ Karena Simetris, maka $L_1 = L_2$.

Sehingga untuk menentukan Luasan total, hitung salah satu luas lalu kalikan 2.

$$L_{II} = \int_0^4\int_{\sqrt{y}}^{2\sqrt{y}} dx\, dy = \int_0^4 x\Big]_{\sqrt{y}}^{2\sqrt{y}} dy$$

$$= \int_0^4 2\sqrt{y}-\sqrt{y}\, dy$$

$$= \int_0^4 \sqrt{y}\, dy$$

$$= \frac{2}{3}y\sqrt{y}\Big]_0^4$$

$$= \frac{2}{3}\left[8\right]$$

$$= \frac{16}{3}$$

$L_{tot} = 2 \times L_{II}$

$= 2 \times \frac{16}{3}$

$= \frac{32}{3}$ satuan luas

d). $r = a(1-\cos\theta)$ terletak di luar lingkaran $r = a$

$r = a(1-\cos\theta)$

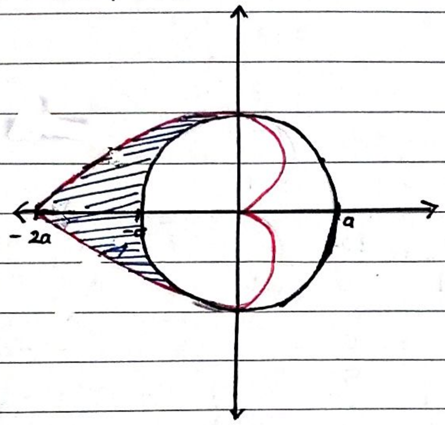
$\theta = 0 \rightarrow r = 0$

$\theta = \pi/2 \rightarrow r = a$

$\theta = \pi \rightarrow r = 2a$

$\theta = 3\pi/2 \rightarrow r = a$

$\theta = 2\pi \rightarrow r = 0$



$L = \iint_D r\,dr\,d\theta$

$= \int_{\pi/2}^{3\pi/2} \int_a^{a(1-\cos\theta)} r\,dr\,d\theta$

$= \int_{\pi/2}^{3\pi/2} \left[\frac{1}{2}r^2\right]_a^{a(1-\cos\theta)} d\theta$

$= \frac{1}{2}\int_{\pi/2}^{3\pi/2} a^2(1-\cos\theta)^2 - a^2\,d\theta$

$= \frac{1}{2}\int_{\pi/2}^{3\pi/2} a^2\left[(1-\cos\theta)^2 - 1^2\right] d\theta$

$= \frac{1}{2}\int_{\pi/2}^{3\pi/2} a^2\left[(1-\cos\theta+1)(1-\cos\theta-1)\right] d\theta$

$= \frac{a^2}{2}\int_{\pi/2}^{3\pi/2} \left[-\cos\theta(2-\cos\theta)\right] d\theta$

$= -\frac{a^2}{2}\int_{\pi/2}^{3\pi/2} 2\cos\theta - \cos^2\theta\,d\theta$

$\blacktriangleright\ 2\int_{\pi/2}^{3\pi/2} \cos\theta = \sin\theta\Big]_{\pi/2}^{3\pi/2}$

$= -1 - 1$

$= -2 \cdot 2 = -4$

$\blacktriangleright\ \int_{\pi/2}^{3\pi/2} \cos^2\theta\,d\theta = \int_{\pi/2}^{3\pi/2} \frac{1}{2}\cos 2\theta + \frac{1}{2}d\theta$

$= \frac{1}{4}\sin 2\theta + \frac{1}{2}\theta\Big]_{\pi/2}^{3\pi/2}$

$= \left(\frac{1}{4}\cdot\sin 3\pi + \frac{1}{2}\cdot\frac{3\pi}{2}\right) - \left(\frac{1}{4}\sin\pi + \frac{1}{2}\frac{\pi}{2}\right)$

$= \left(0 + \frac{3\pi}{4}\right) - \left(0 + \frac{\pi}{4}\right)$

$= \frac{\pi}{2}$

Jawab $= -\frac{a^2}{2}\left(-4 - \frac{\pi}{2}\right)$

$= \frac{a^2}{2}\cdot 4 + \frac{a^2}{2}\cdot\frac{\pi}{2}$

$= 2a^2 + \frac{\pi a^2}{4}$

Jawab : $2a^2 + \frac{\pi a^2}{4}$

g). $x^{2/3} + y^{2/3} = a^{2/3}$

misal : $x = r\cos^3\theta \rightarrow x^{2/3} = r^{2/3}\cos^2\theta$

$y = r\sin^3\theta \rightarrow y^{2/3} = r^{2/3}\sin^2\theta$

$x^{2/3} + y^{2/3} = r^{2/3}\cos^2\theta + r^{2/3}\sin^2\theta = a^{2/3}$

$r^{2/3}\left[\cos^2\theta + \sin^2\theta\right] = a^{2/3}$

$r^{2/3} = a^{2/3}$

$r = a$

✲ Jacobian

$J = \frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix} = \begin{vmatrix} \cos^3\theta & -\frac{3}{2}r\sin 2\theta\cos\theta \\ \sin^3\theta & \frac{3}{2}r\sin 2\theta\sin\theta \end{vmatrix}$

$= \frac{3}{2}r\sin 2\theta\sin\theta\cos^3\theta - \left(-\frac{3}{2}r\sin 2\theta\cos\theta\sin^3\theta\right)$

$= \frac{3}{2}r\sin 2\theta\,\underbrace{\sin\theta\cos\theta}_{\frac{\sin 2\theta}{2}}\cos^2\theta + \frac{3}{2}r\sin 2\theta\,\underbrace{\cos\theta\sin\theta}_{\frac{\sin 2\theta}{2}}\sin^2\theta$

$= \frac{3}{4}r\sin^2 2\theta\cos^2\theta + \frac{3}{4}r\sin^2 2\theta\sin^2\theta$

$= \frac{3}{4}r\sin^2 2\theta\left[\cos^2\theta + \sin^2\theta\right]$

$J = \frac{3}{4}r\sin^2 2\theta$

$\iint_D dx\,dy = \iint_D J\,dr\,d\theta$

✲ Batasan : › Terhadap r : BB : r = 0

BA : r = a

› Terhadap θ : BB : $\theta = 0$

BA : $\theta = 2\pi$

$\int_0^{2\pi}\int_0^a \frac{3}{4}r\sin^2 2\theta\,dr\,d\theta = \frac{3}{4}\int_0^{2\pi}\sin^2 2\theta\left[\left[\frac{1}{2}r^2\right]_0^a\right]d\theta$

$= \frac{3}{4}\int_0^{2\pi}\sin^2 2\theta\left[\frac{1}{2}a^2\right]d\theta$

$= \frac{3}{8}a^2\int_0^{2\pi}\sin^2 2\theta\,d\theta$

✲ $\cos 2\theta = 2\sin^2\theta - 1$

$2\sin^2\theta = \cos 2\theta + 1$

$\sin^2\theta = \frac{\cos 2\theta}{2} + \frac{1}{2}$

$= \frac{3}{8}a^2\int_0^{2\pi}\frac{\cos 4\pi}{2} + \frac{1}{2}\,d\theta$

$= \frac{3a^2}{16}\int_0^{2\pi}(\cos 4\pi + 1)\,d\theta$

$$= \frac{3a^2}{16}\left[\left[\frac{1}{4}\sin 4\theta + \theta\right]_0^{2\pi}\right]$$

$$= \frac{3a^2}{16}\left[\frac{1}{4}(0) + 2\pi - \left(\frac{1}{4}(0) + 0\right)\right]$$

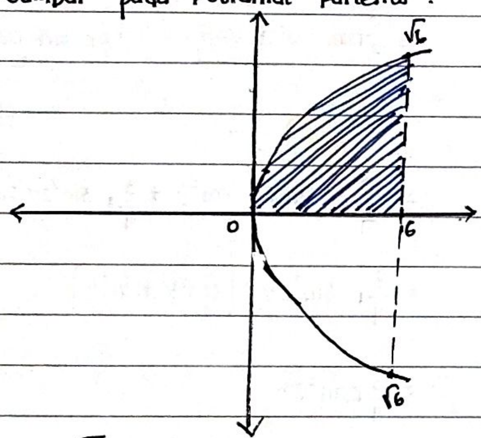
$$= \frac{3a^2}{16} \cdot 2\pi$$

$$= \frac{3a^2}{8}\pi \;//$$

23. Dapatkan koordinat titik berat yang dibatasi oleh kurva-kurva jika density konstan :

a). $y^2 = 6x$; $4y = 0$; $x = 6$

Gambar pada koordinat kartesius :



$$M = \int_0^6\int_0^{\sqrt{6x}} dy\,dx = \int_0^6 y\Big]_0^{\sqrt{6x}} dx$$

$$= \int_0^6 \sqrt{6x}\,dx$$

$$= \sqrt{6}\int_0^6 x^{1/2}\,dx$$

$$= \sqrt{6}\cdot\frac{2}{3}x\sqrt{x}\Big]_0^6$$

$$= \sqrt{6}\cdot\frac{2}{\cancel{3}}\cdot\overset{2}{\cancel{6}}\sqrt{6}$$

$$= 24$$

$$M_y = \int_0^6\int_0^{\sqrt{6x}} x\,dy\,dx = \int_0^6 x\left[\left[y\right]_0^{\sqrt{6x}}\right]dx$$

$$= \int_0^6 x\cdot\sqrt{6x}\,dx$$

$$= \sqrt{6}\int_0^6 x^{3/2}\,dx$$

$$= \sqrt{6}\cdot\frac{2}{5}x^2\sqrt{x}\Big]_0^6$$

$$= \sqrt{6}\cdot\frac{2}{5}\cdot 6^2\cdot\sqrt{6}$$

$$= \frac{432}{5}$$

$$M_x = \int_0^6\int_0^{\sqrt{6x}} y\,dy\,dx = \int_0^6 \frac{1}{2}y^2\Big]_0^{\sqrt{6x}} dx$$

$$= \int_0^6 \cdot\frac{1}{2}(\sqrt{6x})^2\,dx$$

$$= 3\cdot\frac{1}{2}x^2\Big]_0^6$$

$$= \frac{3}{2}\cdot 36$$

$$= 54$$

$$\bar{x} = \frac{M_y}{M} = \frac{432}{5}\cdot\frac{1}{24} \qquad \bar{y} = \frac{M_x}{M} = \frac{54}{24}$$

$$= \frac{18}{5} \qquad = 2\frac{6}{24}$$

$$= 3\frac{3}{5} \qquad = 2\frac{1}{4}$$

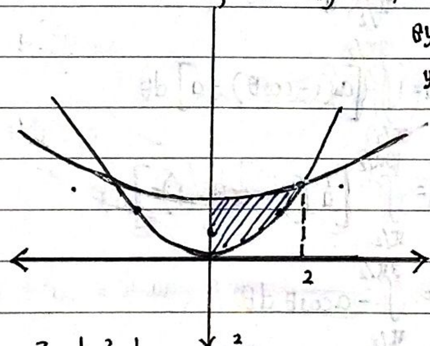
Titik berat $\longrightarrow (\bar{x}, \bar{y}) = \left(3\frac{3}{5}, 2\frac{1}{4}\right)$ //

Jawab = $\left(3\frac{3}{5}, 2\frac{1}{4}\right)$ atau $\left(3\frac{3}{5}, -2\frac{1}{4}\right)$ //

c). $x^2 - 8y + 4 = 0$, $x^2 - 4y = 0$, dan $x = 0$

$$8y = x^2 + 4$$

$$y = \frac{1}{8}x^2 + \frac{1}{2}$$



$$M = \int_0^2\int_{\frac{1}{4}x^2}^{\frac{1}{8}x^2+\frac{1}{2}} dy\,dx = \int_0^2\left[\left[y\right]_{\frac{1}{4}x^2}^{\frac{1}{8}x^2+\frac{1}{2}}\right]dx$$

$$= \int_0^2 \frac{1}{8}x^2 + \frac{1}{2} - \frac{1}{4}x^2\,dx$$

$$= \int_0^2 -\frac{1}{8}x^2 + \frac{1}{2}\,dx$$

$$= \left[-\frac{1}{24}x^3 + \frac{1}{2}x \right]_0^2$$

$$= \left(-\frac{1}{24}(2)^3 + \frac{1}{2}(2) \right) - \left(0 + 0 \right)$$

$$= -\frac{1}{3} + 1$$

$$= \frac{2}{3}$$

$$M_y = \int_0^2 \int_{\frac{1}{4}x^2}^{\frac{1}{8}x^2+\frac{1}{2}} x \, dy\, dx$$

$$= \int_0^2 \left[xy \right]_{\frac{1}{4}x^2}^{\frac{1}{8}x^2+\frac{1}{2}} dx$$

$$= \int_0^2 x \left(\frac{1}{8}x^2 + \frac{1}{2} - \frac{1}{4}x^2 \right) dx$$

$$= \int_0^2 x \left(-\frac{1}{8}x^2 + \frac{1}{2} \right) dx$$

$$= \int_0^2 -\frac{1}{8}x^3 + \frac{1}{2}x \, dx$$

$$= \left[-\frac{1}{32}x^4 + \frac{1}{4}x^2 \right]_0^2$$

$$= -\frac{16}{32} + \frac{4}{4}$$

$$= -\frac{1}{2} + 1$$

$$= \frac{1}{2}$$

$$M_x = \int_0^2 \int_{\frac{1}{4}x^2}^{\frac{1}{8}x^2+\frac{1}{2}} y \, dy\, dx$$

$$= \int_0^2 \frac{1}{2}y^2 \Big]_{\frac{1}{4}x^2}^{\frac{1}{8}x^2+\frac{1}{2}} dx$$

$$= \frac{1}{2}\int_0^2 \left(\frac{1}{8}x^2 + \frac{1}{2} \right)^2 - \left(\frac{1}{4}x^2 \right)^2 dx$$

$$= \frac{1}{2}\int_0^2 \frac{1}{64}x^4 + \frac{1}{4} + \frac{1}{8}x^2 - \frac{x^4}{16} \, dx$$

$$= \frac{1}{2}\int_0^2 -\frac{3}{64}x^4 + \frac{1}{8}x^2 + \frac{1}{4} \, dx$$

$$= \frac{1}{2}\left[\frac{-3}{5.64} . x^5 + \frac{1}{24}x^3 + \frac{1}{4}x \right]_0^2$$

$$= \frac{1}{2}\left[\frac{-3.\cancel{32}}{5.\cancel{64}_2} + \frac{8}{24} + \frac{1}{2} \right]$$

$$= \frac{1}{2}\left[\frac{-3}{10} + \frac{1}{3} + \frac{1}{2} \right]$$

$$= \frac{1}{2}\left[\frac{-9+10+15}{30} \right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{16}^{8}}{30} \right]$$

$$= \frac{4}{15}$$

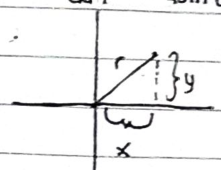
$$\bar{x} = \frac{1}{2} \cdot \frac{3}{2} \qquad \bar{y} = \frac{\cancel{4}^{2}}{\cancel{15}_{5}} \cdot \frac{\cancel{3}}{\cancel{2}}$$

$$= \frac{3}{4} \qquad = \frac{2}{5}$$

Titik Berat : $\left(\frac{3}{4}, \frac{2}{5} \right)$ atau $\left(-\frac{3}{4}, \frac{2}{5} \right)$ //

26). Dengan menggunakan integral lipat dua, dapatkan isi :

f). Persekutuan yang dibatasi oleh $r^2 + z^2 = a^2$ dan $r = a\sin\theta$



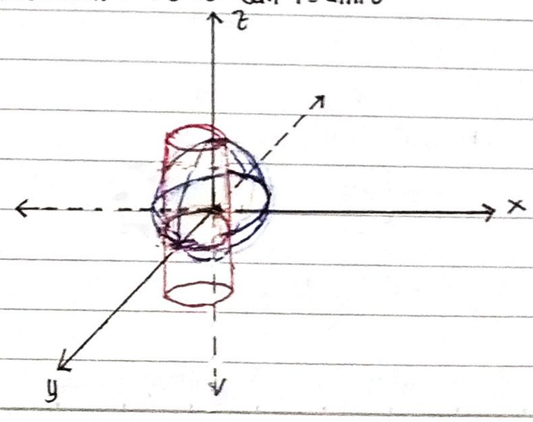
$r^2 = x^2 + y^2$

$\Rightarrow x^2 + y^2 + z^2 = a^2$

- Di bidang $xoy \rightarrow$ lingkaran pusat $(0,0)$ dengan $r = a$
- Di bidang $xoz \rightarrow$ lingkaran pusat $(0,0)$ dengan $r = a$
- Di bidang $yoz \rightarrow$ lingkaran pusat $(0,0)$ dengan $r = a$

Jadi, $x^2 + y^2 + z^2 = a^2$ adalah bola dengan jari-jari adalah a

▶ Persekutuan $r^2 + z^2 = a^2$ dan $r = a\sin\theta$



Rumus Volume persekutuan :

$$\iint_D \int_{-a}^{a} dz\,dy\,dx$$

$$= 2.\iint_D \int_0^a dz\,dy\,dx = 2\iint_D z\Big]_0^a dy\,dx$$

$$= 2a\iint_D dy\,dx$$

$$\Rightarrow 2a\int_0^{2\pi}\int_{\sqrt a}^{a\sin\theta} r\,dr\,d\theta$$

$$= 2a.2\int_0^{\pi}\int_{\sqrt a}^{a\sin\theta} r\,dr\,d\theta$$

$$= 4a\int_0^{\pi}\int_{\sqrt a}^{a\sin\theta} r\,dr\,d\theta$$

$$= 4a\int_0^{\pi} \frac12 r^2\Big]_{\sqrt a}^{a\sin\theta} d\theta$$

$$= \cancel{4}^{2}a\,\frac{1}{\cancel{2}}\int_0^{\pi} a^2\sin^2\theta - a\,d\theta$$

$$= 2a\int_0^{\pi} a(a\sin^2\theta - 1)\,d\theta$$

$$= 2a^2\int_0^{\pi} a\sin^2\theta - 1\,d\theta$$

$$= 2a^2\left[\int_0^{\pi} a\sin^2\theta\,d\theta - \int_0^{\pi} d\theta\right]$$

* $\int_0^{\pi} a\sin^2\theta\,d\theta = a\int_0^{\pi} \frac12 - \frac{\cos 2\theta}{2}\,d\theta$

$$= \frac a2\int_0^{\pi} 1-\cos 2\theta\,d\theta$$

$$= \frac a2\left[\theta\Big]_0^{\pi} - \frac12\sin 2\theta\Big]_0^{\pi}\right]$$

$$= \frac a2\left[\pi - 0\right]$$

$$= \frac{a\pi}{2}$$

* $\int_0^{\pi} d\theta = \theta\Big]_0^{\pi} = \pi$

$$\Rightarrow 2a^2\int_0^{\pi} \sin^2\theta\,d\theta - \int_0^{\pi} d\theta$$

$$= 2a^2\left[\frac{a\pi}{2} - \pi\right]$$

$$= 2a^2\pi\left(\frac a2 - 1\right)$$

$$= a^2\pi(a-2) \text{ satuan volume}$$

g). Hitung volume benda yang dibatasi bidang $z = 3x - y$ yang terletak di atas prisma jajaran genjang dibatasi oleh garis-garis $x - y = -1$, $x - y = 1$, $x - 3y = -9$ dan $x - 3y = -5$

$$\iint_D\int_0^{3x-y} dz\,dy\,dx = \iint_D z\Big]_0^{3x-y} dy\,dx$$

$$= \iint_D (3x-y)\,dy\,dx$$

misal : $u = x - y$ $\quad v = x - 3y$

$u_x = 1$ $\quad v_x = 1$

$u_y = -1$ $\quad v_y = -3$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & -1 \\ 1 & -3 \end{vmatrix}} = \frac{1}{(-3-(-1))}$$

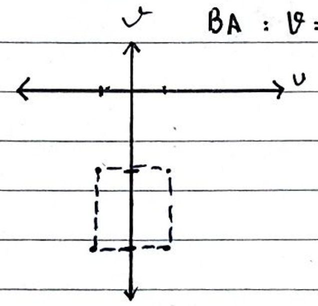
$$= \frac{1}{-2}$$

Batas - Batas : -> Terhadap u :

BA : $u = 1$; BB : $u = -1$

-> Terhadap v :

BA : $v = -5$; BB : $v = -9$



$$\begin{array}{l|c|l} u = x - y & \times 4 & 4u = 4x - 4y \\ v = x - 3y & \times 1 & v = x - 3y \\ \hline & & 4u - v = 3x - y \end{array}$$

Luas : $\int_{-1}^{1}\int_{-9}^{-5} (4u - v)\left|-\frac12\right| dv\,du$

$$= \frac12\int_{-1}^{1}\left[4uv - \frac12 v^2\right]_{-9}^{-5} du$$

$$= \frac12\int_{-1}^{1}\left[\left(-20u - \frac{25}{2}\right) - \left(-36u - \frac{81}{2}\right)\right] du$$

$$= \frac12\int_{-1}^{1}\left[16u + \frac{56}{2}\right] du$$

$$= \frac12\int_{-1}^{1}\left[16u + 28\right] du$$

$$= \frac12\left[8u^2 + 28u\right]_{-1}^{1}$$

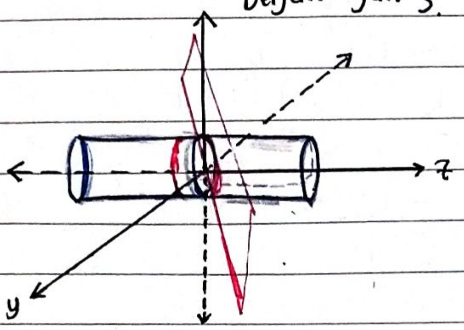
$$= \frac12\left[(8+28) - (8-28)\right]$$

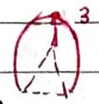
$$= \frac12\left[56\right] = 28 \text{ satuan volume}$$

h). Hitung volume dari benda dibatasi oleh bidang xoy, bidang yoz, bidang $x = y$, dan bidang $z = \sqrt{9-y^2}$

$z^2 = 9 - y^2$

$y^2 + z^2 = 9 \longrightarrow$ tabung yang memanjang sepanjang sumbu x dan berjari-jari 3.

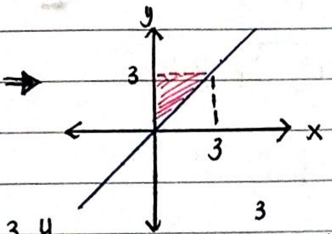




Volume : $\iint_D \int_0^3 dz\, dy\, dx = \iint_D z\Big]_0^3 dy\, dx$

$= \iint_D 3\, dy\, dx$

$\Rightarrow$



$$\int_0^3\int_0^y 3\, dx\, dy = 3\int_0^3 x\Big]_0^y dy$$

$$= 3\int_0^3 y\, dy$$

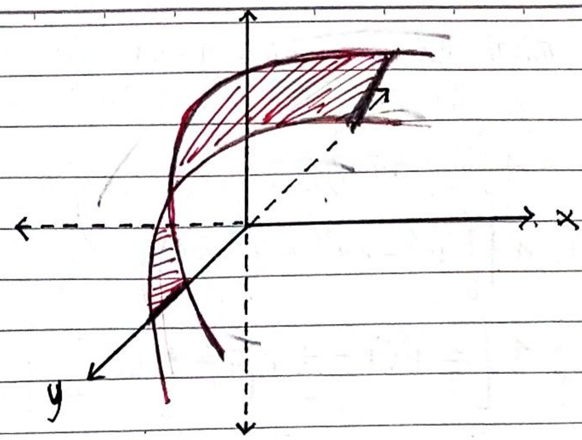
$$= 3 \cdot \frac{1}{2}y^2\Big]_0^3$$

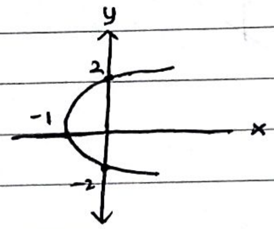
$$= \frac{3}{2} \cdot 9$$

$$= \frac{27}{2} \text{ satuan volume}$$

//

25 Dapatkan momen inersia dari dataran yang dibatasi oleh $y^2 = 4x + 4$, sumbu y, terhadap sumbu/garis tegak lurus bidang xy di titik 0





$\Rightarrow I_x = \iint_A y^2\, dx\, dy$

$$= 2\int_{-1}^{0}\int_0^{\sqrt{4x+4}} y^2\, dy\, dx$$

$$= 2 \cdot \int_{-1}^{0} \frac{1}{3}y^3\Big]_0^{\sqrt{4x+4}} dx$$

$$= \frac{2}{3}\int_{-1}^{0} 4x+4\sqrt{4x+4}\, dx$$

Misal : $u = 4x + 4$ BA : $u = 4(0) + 4$

$du = 4\, dx$ $= 4$

$dx = \frac{du}{4}$ BB : $u = 4(-1) + 4$

$= 0$

$$= \frac{2}{3}\int_0^4 u^{3/2}\frac{du}{4}$$

$$= \frac{1}{6} \cdot \frac{2}{5}u^2\sqrt{u}\Big]_0^4$$

$$= \frac{2}{30}\Big[16.2 - 0\Big]$$

$$= \frac{32}{15}$$

$\Rightarrow I_y = \iint_A x^2\, dy\, dx$

$$= 2\int_{-1}^{0}\int_0^{\sqrt{4x+4}} x^2\, dy\, dx$$

$$= 2\int_{-1}^{0} x^2 y\Big]_0^{\sqrt{4x+4}} dx$$

$$= 2\int_{-1}^{0} x^2\left(\sqrt{4x+4}\right) dx$$

misal: $u = x+1$ BA : $u = 0+1 = 1$

$du = dx$ BB : $u = (-1)+1 = 0$

$= 4\int_0^1 (u^2-2u+1)\sqrt{u}\,du$

$= 4\int_0^1 u^{5/2} - 2u^{3/2} + u^{1/2}\,du$

$= 4\left[\frac{2}{7}u^3\sqrt{u} - \frac{4}{5}u^2\sqrt{u} + \frac{2}{3}u\sqrt{u}\right]_0^1$

$= 4\left[\frac{2}{7} - \frac{4}{5} + \frac{2}{3}\right]$

$= 4\left[\frac{30-84+70}{105}\right]$

$= \frac{4.16}{105}$

$= \frac{64}{105}$

$I = J_x + J_y$

$= \frac{32}{15} + \frac{64}{105}$

$= \frac{224+64}{105}$

$= \frac{288}{105}$

$= \frac{96}{35}$ //

28. Dapatkan Luas permukaan bidang lengkung dari:

b). Bagian Bola $x^2+y^2+z^2=36$ dalam tabung $x^2+y^2=6y$

misal : $x = r\cos\theta \rightarrow x^2 = r^2\cos^2\theta$

$y = r\sin\theta \rightarrow y^2 = r^2\sin^2\theta$

$x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2(\cos^2\theta + \sin^2\theta)$

$= r^2(1)$

$= r^2$

* Jacobian : $J = \frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix}$

$= \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$

$= r(\cos^2\theta + \sin^2\theta)$

$= r$

Persamaan baru :

$x^2+y^2+z^2 = 36$

$r^2 + z^2 = 36$

$z = \sqrt{36-r^2}$

$x^2+y^2 = 6y$

$r^2 = 6.r\sin\theta$

$r = 6\sin\theta$

* $z = (36-r^2)^{1/2}$

$z_r = \frac{1}{\cancel{2}}.(36-r^2)^{-1/2}.-\cancel{2}r$

$= \frac{-r}{\sqrt{36-r^2}}$

$z_\theta = 0$

$K = \iint_A \sqrt{1+(z_r)^2+\frac{1}{r^2}(z_\theta)^2}\, r\,dr\,d\theta$

$= \int_0^\pi\int_0^{6\sin\theta} \sqrt{1+\frac{r^2}{36-r^2}}\, r\,dr\,d\theta$

$= \int_0^\pi\int_0^{6\sin\theta} \sqrt{\frac{36-r^2+r^2}{36-r^2}}\, r\,dr\,d\theta$

$= \int_0^\pi\int_0^{6\sin\theta} \frac{6r}{\sqrt{36-r^2}}\,dr\,d\theta$

misal : $36-r^2 = p$ BA: $p = 36-36\sin^2\theta$

$-2r\,dr = dp$ BB: $p = 36-0$

$dr = \frac{-dp}{2r}$ $= 36$

$= \int_0^\pi\int_{36}^{36-36\sin^2\theta} \frac{3\cancel{6r}}{\sqrt{p}}.\frac{-dp}{\cancel{2r}}\,d\theta$

$= -3\int_0^\pi\int_{36}^{36-36\sin^2\theta} p^{-1/2}\,dp\,d\theta$

$= -3\int_0^\pi 2.\sqrt{p}\Big]_{36}^{36-36\sin^2\theta} d\theta$

$= -6\int_0^\pi \left[\sqrt{36(1-\sin^2\theta)} - \sqrt{36}\right] d\theta$

$= -6\int_0^\pi \left[6\left(\sqrt{\cos^2\theta} - 1\right)\right] d\theta$

$= -36\int_0^\pi [\cos\theta - 1]\,d\theta$

$= -36\left[\sin\theta - \theta\right]_0^\pi$

$= -36\left[(0-\pi) - (0-0)\right]$

$= 36\pi$

Jawab $= 36\pi$ //

1 d). Bagian bola $x^2+y^2+z^2=25$ di antara bidang $z=2$ dan $z=4$

misal : $x = r\cos\theta \rightarrow x^2 = r^2\cos^2\theta$

$y = r\sin\theta \rightarrow y^2 = r^2\sin^2\theta$

$$x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$$
$$= r^2\left[\cos^2\theta+\sin^2\theta\right]$$
$$= r^2\,(1)$$
$$= r^2$$

※ Jacobian : $J = \dfrac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix}$

$$= \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$
$$= r\cos^2\theta - (-r\sin^2\theta)$$
$$= r(\cos^2\theta+\sin^2\theta)$$
$$= r$$

Persamaan Baru :

$$x^2+y^2+z^2 = 25$$
$$r^2+z^2 = 25$$
$$z^2 = 25-r^2$$
$$z = \sqrt{25-r^2}$$

$$z_r = \frac{1}{2}(25-r^2)^{-1/2}.(-2r)$$
$$= \frac{-r}{\sqrt{25-r^2}}$$

$z_\theta = 0$

※ Batas $z=2$ dan $z=4$

$z = \sqrt{25-r^2}$

- $z=2 \longrightarrow 2 = \sqrt{25-r^2}$

$$4 = 25-r^2$$
$$r^2 = 21$$
$$r = \pm\sqrt{21}$$

- $z=4 \longrightarrow 4 = \sqrt{25-r^2}$

$$16 = 25-r^2$$
$$r^2 = 9$$
$$r = \pm 3$$

BB = 3

BA = $\sqrt{21}$

$$K = \int_0^{2\pi}\int_3^{\sqrt{21}} \sqrt{1+\frac{r^2}{25-r^2}}\; r\,dr\,d\theta$$
$$= \int_0^{2\pi}\int_3^{\sqrt{21}} r\sqrt{\frac{25-r^2+r^2}{25-r^2}}\,dr\,d\theta$$
$$= \int_0^{2\pi}\int_3^{\sqrt{21}} r\sqrt{\frac{25}{25-r^2}}\,dr\,d\theta$$
$$= 5\int_0^{2\pi}\int_3^{\sqrt{21}} \frac{r}{\sqrt{25-r^2}}\,dr\,d\theta$$

misal : $u = 25-r^2$ BA : $u = 25-21 = 4$

$du = -2r\,dr$ BB : $u = 25-9 = 16$

$dr = \dfrac{-du}{2r}$

$$= \frac{-5}{2}\int_0^{2\pi}\int_{16}^{4} \frac{\cancel{r}}{\sqrt{u}}.\frac{du}{\cancel{r}}\,d\theta$$
$$= \frac{-5}{2}\int_0^{2\pi} 2\sqrt{u}\Big]_{16}^{4}\,d\theta$$
$$= \frac{-5}{2}\int_0^{2\pi} 2\sqrt{4}-2\sqrt{16}\,d\theta$$
$$= -5\int_0^{2\pi} -2\,d\theta$$
$$= 10\left[\theta\Big]_0^{2\pi}\right]$$
$$= 10.2\pi$$
$$= 20\pi$$